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USING SYNTHETIC BEACONS

by

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ANISOPLANATISM IN ADAPTIVE OPTICS
USING SYNTHETIC BEACONS

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ABSTRACT

The residual phase structure function of a main laser beam with synthetic beacons is derived. The effects of focal anisoplanatism on the compensation efficiency of adaptive optical system using synthetic beacons are discussed.
Key Words: synthetic beacon, adaptive optics, focal anisoplanatism.

1. Introduction

It is well known that one of the key problems in applying an adaptive optical system in turbulence effect phase compensation for laser beams propagated through the atmosphere is the light source to be selected for the beacons. Since an appropriate star cannot always be found in the isoplanatic angle to serve as a light source for beacons, researchers place their hopes on synthetic beacons, such as fabricating light sources for synthetic beacons using laser atmospheric back-scattering or on sodium-layers[1,2]. In doing so, a new problem arises, namely: a synthetic beacon is a "point" light source over limited distances, and the overlapping region of the synthetic beacon and the main laser beam is a cone, beyond which the phase information of the main laser beam is not carried by the beacon light. In other words, the region outside the cone forms an anisoplanatic region--focal anisoplanatism[3].

This paper first derives the residual phase structure function between the synthetic beacon and the main laser beam, and then analyzes the effects of focal anisoplanatism on the compensation efficiency of adaptive optical system using synthetic beacons.

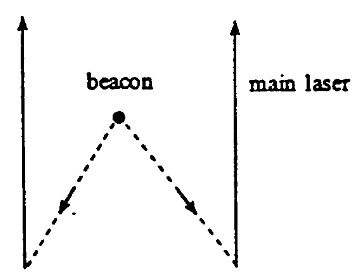

Fig. 1. Diagram of synthetic beacon

2. Residual Phase Structure Function

The main laser beam phase is marked with $\Phi_m(r)$ and the beacon light phase is denoted with $\Phi_b(r)$ as shown in Fig. 1. They can be, respectively, written as

$$\Phi_m(r) = k\int_0^H \delta n(r, z)dz \tag{1}$$

$$\Phi_b(r) = k\int_0^{H_b} \delta n[(1-\frac{z}{H_b})r, z]dz \tag{2}$$

Here, let the main laser beam and the beacon light share the same wavelength; H_b in the equation is the height of the beacon light source, H is the thickness of the atmosphere or the distance of main laser beam propagation through the atmosphere, $k=2\pi/\lambda$ is the number of laser waves and λ is wavelength. Thus, the residual phase between the synthetic beacon and main laser beam is:

$$\Delta\Phi(r)=k\int_0^{H_b}\{\delta n(r,z)-\delta n[(1-\frac{z}{H_b})r,z]\}dz-k\int_{H_b}^{H}\delta n(r,z)dz \qquad (3)$$

where δn is the fluctuation in refractivity. Under general conditions, H_b is sufficiently high so that the second term on the right-hand side of Eq. (3) can be neglected. As a result, the residual phase structure function can be derived as follows:

$$\begin{aligned} D_{\Delta\varphi}(\bar{\rho},r)=k^2\int_0^H dz\int_{-H}^{H}dh\{&-D_n(0,h)+D_n(\bar{\rho},h)+2D_n(\frac{z}{H_b}r,h)-D_n(\bar{\rho}+\frac{z}{H_b}r,h)\\ &-D_n(\bar{\rho}-\frac{z}{H_b}r,h)-D_n(\frac{h}{H_b}r,h)+\frac{1}{2}D_n[(1-\frac{z}{H_b})\bar{\rho}-\frac{h}{H_b}r,h]\\ &+\frac{1}{2}D_n[(1-\frac{z}{H_b})\bar{\rho}+\frac{h}{H_b}r,h]\} \end{aligned} \qquad (4)$$

where $\bar{\rho}=r_2-r_1$, $h=z_2-z_1$, $D_n(\bar{\rho},z_1,z_2)=c_n^2(\frac{z_1+z_2}{2})[|\bar{\rho}|^2+h^2]^{1/3}$ is the refractivity fluctuation structure function; (r_1,z_1), $(r_2\ z_2)$ are the coordinates of two points (r,z); r is the coordinate of laser beam cross section, z is the coordinate along the direction of laser beam propagation. When $|\bar{\rho}|<L_0$, according to reference [4], the integration limit of h can be extended to $(-\infty,\infty)$, while L_0 is the turbulence external scale. Taking into consideration $\frac{|r|}{H_b}\ll 1$, $|\frac{1}{H}(1-\frac{z}{H})r\cdot\bar{\rho}|\ll|(1-\frac{z}{H})\bar{\rho}|$, then

$$\begin{cases}(1+\frac{|r|^2}{H_b^2})h^2\approx h^2\\ |(1-\frac{z}{H_b})\bar{\rho}\pm\frac{h}{H_b}r|^2+h^2\approx|(1-\frac{z}{H_b})\bar{\rho}|^2+h^2\end{cases} \qquad (5)$$

From Eq. (4), the following can be obtained:

$$\begin{aligned} D_{\Delta\varphi}(\bar{\rho},r)=2.91k^2\int_0^H dz\,c_n^2(z)[&|\bar{\rho}|^{5/3}+2|\frac{z}{H_b}r|^{5/3}-|\bar{\rho}+\frac{z}{H_b}r|^{5/3}\\ &-|\bar{\rho}-\frac{z}{H_b}r|^{5/3}+|(1-\frac{z}{H_b})\bar{\rho}|^{5/3}] \end{aligned} \qquad (6)$$

Obviously, when the adaptive optical system using synthetic beacons with a limited height is used in phase compensation for laser beam atmospheric propagation, the residual phase structure function not only is a function of $|\bar{\rho}|=|r_2-r_1|$, but also is related to the location coordinate of the laser beam cross section. Actually, it can be seen directly from Fig. 1 that the closer to the center of the laser beam, the smaller the residual phase can be, or vice versa, the larger it will be. By comparing Eq. (6) with the angular anisoplanatism in references [4] and [5], it is found that the two agree with each other in form, but the difference is that the included angle between the main laser beam and the beacon light in Eq. (6) is a function of the coordinate on the laser beam cross section r, i.e., $\theta(r)=|\frac{r}{H_b}|$.

3. Effect of Focal Anisoplanatism on Compensation Efficiency

The Strehl ratio is generally used to describe the compensation efficiency of an adaptive optical system. According to the definition of the Strehl ratio,

$$S_t=(\frac{1}{4}\pi D^2)^{-1}\langle|\int u(r)w(r)dr|^2\rangle\Big/\int|u(r)|^2dr \tag{7}$$

where D is aperture, u(r) is optical field function, w(r) is aperture function, < > is the system integration average. Under general conditions,

$$w(r)=\begin{cases}1, & |r|\leqslant D/2\\ 0, & |r|>D/2\end{cases} \tag{8}$$

$\int|u(r)|^2\mathrm{d}r=\pi R^2I_0$, I_0 , where I_0 is light intensity on optical axis; for plane waves D=2R; for the Gaussian beam $D=2\sqrt{2}R$. Let $u_1(r)=u(r)/\sqrt{I_0}$, $K_1(\bar{\rho},r)=\frac{1}{\pi R^2}w(r-\bar{\rho})w(r)$. Thus, Eq. (7) becomes

$$S_t=(\frac{1}{4}\pi D^2)^{-1}\iint K_1(\bar{\rho},r)\mathrm{MCF}(\bar{\rho},r)\mathrm{d}\bar{\rho}\,\mathrm{d}r \tag{9}$$

where $MCF(\rho,r)=\langle u_1(\bar{\rho},r)u_1^*(r)\rangle$ is the intercorrelation function of optical field; the superscript "*" denotes composite conjugation. When the effect of amplitude fluctuation is neglected,

$$MCF(\bar{\rho}, r)=\exp[-\frac{1}{2} D_\varphi(\bar{\rho}, r)] \tag{10}$$

where $D_\varphi(\bar{\rho},r)$ is the phase structure function. When the phase fluctuation is isotropic, the phase structure function is only a function of $|\bar{\rho}|$. In this case, the formula for calculating the Strehl ratio is a general expression[6]

$$S_t=(\frac{1}{4}\pi D^2)^{-1}\int d\bar{\rho}\, k(\bar{\rho})MCF(\bar{\rho}) \tag{11}$$

where $K(\bar{\rho})=\frac{1}{\pi R^2}\int drw(r-\bar{\rho})w(r)=\frac{2}{\pi}[\arccos\frac{|\bar{\rho}|}{D}-\frac{|\bar{\rho}|}{D}\sqrt{1-\frac{|\bar{\rho}|^2}{D^2}}]$. In focal anisoplanatism, the phase structure function is a function of and r. Therefore, in computing the Strehl ratio, it is necessary to adopt Eqs. (9) and (10) and carry out numerical integration. Here, we discuss only one typical case, i.e., $H_b>H$. In fact, the sodium-layer synthetic beacon can satisfy this condition, because the sodium layer has a height of approximately 80-100km, while the effect of atmospheric turbulence on laser beam propagation basically concentrates in a range from 0 to 10km, or no more than 30km even if high-altitude turbulence H is taken into consideration. Moreover, c_n generally decreases with increase in altitude. Thus, Eq. (6) can be simplified as

$$D_{\Delta\varphi}(r)=2.91R^2\int_0^H dzc_n^2(z)[2|\frac{z}{H_b}r|^{5/3}]=2|\frac{r}{H_b}|^{5/3}/\theta_0^{5/3} \tag{12}$$

where θ_0 is the isoplanatic angle. In this case, it is easy to derive the computation formula for the Strehl ratio from Eqs. (10) and (9)

$$S_t=(\frac{1}{4}\pi D^2)^{-1}\int dr\exp[-|\frac{r}{H_b}|^{5/3}/\theta_0^{5/3}] \tag{13}$$

It can be seen from Eq. (13) that: (1) The larger the transmitting aperture D, the larger the anisoplanatic region can be and the poorer the compensation efficiency of the adaptive optical system using synthetic beacons, which is just the opposite to angular anisoplanatism; (2) The larger the height of the synthetic beacon, the smaller the $\theta(r)=|r|/H_b$, and the better the compensation efficiency or vice versa, the poorer the compensation efficiency.

4. Conclusions

The wave front residual variance can also be used to analyze the focal anisoplanatism, i.e.,

$$\begin{cases} S_t \approx \exp(-\sigma^2_{\Delta\varphi}) \\ \sigma^2_{\Delta\varphi} = (\frac{1}{4}\pi D^2)^{-1} \int \langle \Delta\varphi^2(r) \rangle \mathrm{d}r \end{cases} \tag{14}$$

It is not difficult to derive a similar method as described in section 2

$$\sigma^2_{\Delta\varphi} = \frac{6}{11} (\frac{\theta}{\theta_0})^{5/3} \tag{15}$$

where $\theta=D/2H_b$. As a matter of fact, it is necessary to meet the condition $\sigma^2_{\Delta\varphi}<1.0$ at this moment. Under similar approximation conditions, a result similar to that described above can also be obtained from Eq. (13). For instance, suppose the transmitting aperture D=1.0m, H_b=100km, θ is approximately 5μrad; for appproximately1.0μm laser atmospheric propagation, θ and the atmospheric isoplanatic angle θ_0 are in the same order of magnitude, indicating that the focal anisoplanatism of synthetic beacons is a critical factor that restricts the compensation efficiency. If Rayleigh back-scattering is used as a beacon, then the problem will be even more serious. More than this, H_b is rather small at this moment, around 20km or less. In such case, the atmospheric turbulence information higher than H_b also cannot be obtained from the beacon.

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